3. STELLAR ATMOSPHERES

3.1 The Radiation Field - Basic Definitions (2B12)

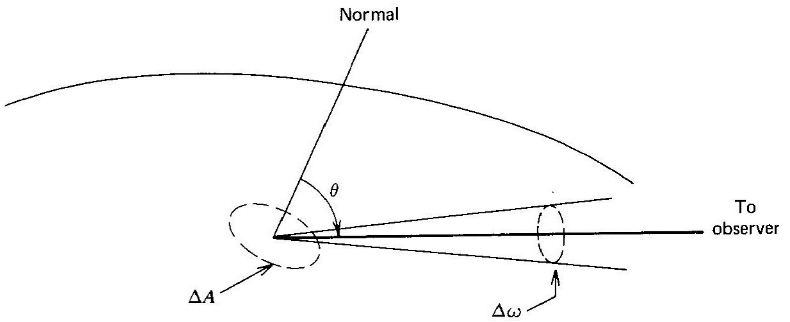

Fig. 5.1. The geometrical portion of the definition of specific intensity is illustrated here. The increment of area, ΔA, is seen foreshortened by $\cos\theta$, where θ is the angle of view from the normal to the surface. An increment of solid angle, $\Delta\omega$, is shown.

[From Gray (1995)]

3.1.1 Specific Intensity (J $\mathrm{m^{-2}}$ $\mathrm{s^{-1}}$ $\mathrm{Hz^{-1}\,str^{-1}}$)

Consider a radiating surface as shown above. The specific intensity is the monochromatic intensity or brightness of a beam of radiation observed from some direction (θ, ϕ) at a point on the surface and is defined by

$$I_\nu = \frac{dE_\nu}{\cos\theta\, dA\, dt\, d\nu\, d\omega} \tag{3.1}$$

where θ is the angle between the beam and the normal to the surface, and dE_ν is the amount of energy.

Polar coords: $d\omega = \sin\theta d\theta d\phi$. θ runs from 0 to π and ϕ from 0 to 2π. $\theta = 0$ along the axis of symmetry (e.g. polar axis of a rotating star). Often set $\mu = \cos\theta$. In the plane-parallel case, there is azimuthal symmetry and I_ν is then only a function of θ (or μ). The integral of ϕ over $d\omega$ is 2π.

I_ν is only measurable directly if the surface is resolved (only possible for the sun). It is usually obtained from the equation of radiative transfer (see later).

To obtain quantities which relate the radiation field to observables, we need to integrate over solid angle to give angular averages.

3.1.2 Moments of the Radiation Field - angular averages

(1) Mean Intensity (J $\mathrm{m^{-2}\,s^{-1}\,Hz^{-1}}$)

J_ν is I_ν integrated over all solid angles and it represents the brightness irrespective of direction i.e. it is the directional average of the specific intensity.

$$J_\nu = \frac{\oint I_\nu \, d\omega}{\oint d\omega} = \frac{1}{4\pi} \oint I_\nu \, d\omega \tag{3.2}$$

where $\oint d\omega = 4\pi$. For plane-parallel stellar atmosphere layers (i.e. no curvature):

$$d\omega = \sin\theta d\theta d\phi \tag{3.3}$$

and then

$$J_\nu = \frac{1}{2} \int_{-1}^{1} I_\nu \, d\mu \tag{3.4}$$

(2) Flux (J m^{-2} s^{-1} Hz^{-1})

$\mathcal{F}_\nu$ is the integral of I_ν and μ over all solid angles. It is a measure of the net flow of energy perpendicular to dA whose normal makes an angle θ with respect to the observer, in time dt in spectral range $d\nu$. Flux is a vector. It depends on θ. Intensity is a scalar.

$$\mathcal{F}_\nu = \oint I_\nu \cos\theta d\omega = 2\pi \int_{-1}^{1} I_\nu \mu \, d\mu \tag{3.5}$$

Flux is often separated into two parts – an ingoing and an outgoing part: $\mathcal{F}_\nu = \mathcal{F}_\nu^{\rm in} + \mathcal{F}_\nu^{\rm out}$. When the radiation is isotropic, $\mathcal{F}_\nu = 0$ since $\mathcal{F}_\nu^{\rm in} = -\mathcal{F}_\nu^{\rm out}$. For a stellar atmosphere $\mathcal{F}_\nu^{\rm in} = 0$. The flux emitted by a stellar disk (i.e. over one hemisphere with $\theta = 0$ to $\pi/2$) $\mathcal{F}_\nu = \pi I_\nu$. Defining the astrophysical flux as $F_\nu = \mathcal{F}_\nu/\pi$, then $F_\nu = I_\nu$ for a stellar disk.

The Eddington flux $H_\nu = \frac{1}{4\pi}\mathcal{F}_\nu$ and thus (by analogy to J_ν)

$$H_\nu = \frac{1}{2} \int_{-1}^{1} I_\nu \mu \, d\mu \tag{3.6}$$

(3) The Second Order Moment K_ν (J m^{-2} s^{-1} Hz^{-1})

K_ν is often called the "K integral" and physically, it is related to the radiation pressure.

$$K_\nu = \frac{1}{4\pi} \oint I_\nu \cos^2\theta \, d\omega = \frac{1}{2} \int_{-1}^{1} I_\nu \mu^2 \, d\mu \tag{3.7}$$

The radiation pressure P_ν is given by

$$P_\nu = \frac{1}{c} \oint I_\nu \cos^2\theta \, d\omega = \frac{4\pi}{c} K_\nu \tag{3.8}$$

3.1.3 Energy Density U_ν

$$U_\nu = \frac{1}{c} \oint I_\nu \, d\omega = \frac{4\pi}{c} J_\nu \tag{3.9}$$

3.1.4 Isotropic Radiation Field (I_ν is not a function of μ)

$$J_\nu = I_\nu \tag{3.10}$$

$$\mathcal{F}_\nu = 0 \tag{3.11}$$

$$K_\nu = \frac{1}{3}\, J_\nu \tag{3.12}$$

For an isotropic, time-independent radiation field given by the Planck function $B_\nu(T)$

$$I_\nu = J_\nu = B_\nu(T). \tag{3.13}$$

The energy density

$$U_\nu = \frac{4\pi}{c}\, B_\nu(T) \tag{3.14}$$

The total energy density

$$U = \int \frac{4\pi}{c}\, B_\nu(T) = aT^4 \tag{3.15}$$

where a is the radiation constant $= 7.56 \times 10^{-16}\ \mathrm{J\,m^{-3}\,K^{-4}}$.

From eqns. (3.8) and (3.12),(3.13) and (3.14) the radiation pressure

$$P = \int P_\nu\ d\nu = \int \frac{1}{3} U_\nu = \frac{1}{3} aT^4 \tag{3.16}$$

3.2 Interaction of Radiation and Matter (2B12)

As the beam I_ν passes through a medium, photons can be created or destroyed. The net change in the energy gives the change in intensity (eqn 3.1). The study of the change in intensity is called "radiative transfer".

3.2.1 Extinction Coefficient and Optical Depth

The energy removed from the radiation field as it passes through a medium of length dx is given by

$$dI_\nu = -\chi_\nu I_\nu\ dx \tag{3.17}$$

where χ_ν is the extinction coefficient or opacity (units $\mathrm{m^{-1}}$). The I_ν on the right hand side is necessary because there has to be some photons to destroy.

$$\chi_\nu = \kappa_\nu + \sigma_\nu \tag{3.18}$$

where
$\kappa_\nu =$ absorption coefficient and a photon is destroyed by being absorbed by the material.
$\sigma_\nu =$ scattering coefficient and a photon is scattered out of the beam i.e. energy is removed but not destroyed.

If σ_ν is isotropic and equal to zero (i.e. what is scattered out is scattered back in to the beam), then $\chi_\nu = \kappa_\nu$.

Confusingly, the extinction coefficient is sometimes written as the mass extinction coefficient and then has units of m^2 kg because it is multiplied by the density ρ. With the definition given in (3.17), the mean free path is simply $1/\chi_\nu$.

Optical depth

$$d\tau_\nu = \chi_\nu \; dx \tag{3.19}$$

$$dI_\nu = -I_\nu d\tau_\nu \tag{3.20}$$

$$I_\nu = I_\nu^0 \exp(-\tau_\nu) \tag{3.21}$$

This is an extinction law and measures how far we can see into a stellar atmosphere.

3.2.2 Emission Coefficient and Source Function

The energy added to the radiation field

$$dI_\nu = j_\nu \; dx \tag{3.22}$$

where j_ν is the emission coefficient for thermal emission (creation of photons).

The source function is defined as

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \tag{3.23}$$

In steady state thermal equilibrium (no net gain or loss of energy) and with pure emission and absorption:

$$j_\nu = \kappa_\nu \; I_\nu \tag{3.24}$$

(Kirchoff's Law)

and with $I_\nu = B_\nu(T)$

$$j_\nu = \kappa_\nu B_\nu(T) \tag{3.25}$$

(Kirchoff-Planck relation)

and thus

$$S_\nu = B_\nu(T) \tag{3.26}$$