Lecture 1 (i.e. week 1)

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- Welcome + personal introduction

- Course structure

→ lectures: 3h/week
Tue 16.00 - 17.00
Thu 13.00 - 14.00
Thu 15.00 - 16.00

→ tutorials: 1h/week (4 groups) Either Mon. or Tue.

→ Attendence at the lectures is compulsory ⇒ [sign in] ⇐

→ On Friday I'll give a set of exercises on the material covered during the week. Return them by Thursday 4 pm

→ The exerc. are discussed during the tutorials + I'll try to answer any questions you have on Quantum Mechanics.

→ TUTORIALS NEXT WEEK

- Assessment
15% weekly homework ← IMPORTANT!
5% mid-term test
+ 80% Final test

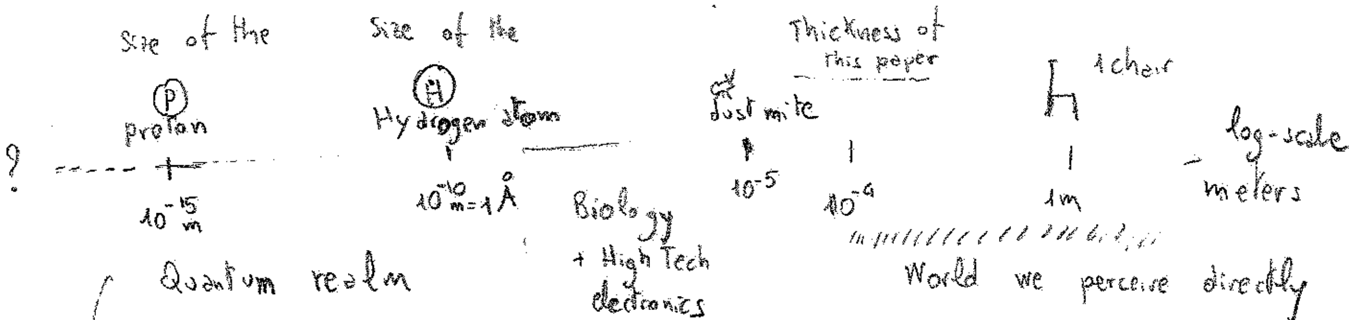
- References & notes: I'll be writing on the blackboard.
Please stop me anytime if
  - I'm too fast
  - I make a mistake
  - You've a question

→ books
- Young & Freedman
- Krane
- R. Feynman + R. Leighton + M. Sands "The Feynman lectures on Physics" special room

ANY QUESTIONS?

Aim of the course

- The .. is to introduce you to the strange and surprising world of Quantum Mechanics, that is the set of rules that describes the behaviour of very small objects. (like atoms, protons, electrons..)



It turns out that objects leaving in the quantum realm share some features we usually associate to particles and some other features we associate to waves... which means that we can not faithfully picture the quantum-objects neither as very small ball nor as a wave. This is the source of "strageness" of Quantum Mechanics.

- The course will deal mainly with physical concepts and I will keep the mathematics required as basic as possible. This means that you will need to attend QMA and QMB to study in detail interesting (but complicated) systems like the structure of the atom.

Outline of the course

matter

- Mini-review of Newton's laws and their limit of validity $\xrightarrow{\text{application}}$ Derive the specific heat of various gas (A)

light

- Waves. Phaenomena of interference and diffractions → Photoelectric effect (B)
  → black body radiation (C)

The experiments for (A), (B) and (C) give results that are not in agreement with Newton's law (matter) or Maxwell eqs. (light)

⇝ This was source of great concern for physicists at the end of XIX century.

The reason of this discrepancy is due to the strange behaviour of atoms and other quantum objects (in (A), (B) and (C) one can see macroscopical manifestations of Q.M.: this is how/why Q.M. was discovered/invented).

- Duality particle / wave
- Heisenberg uncertainty principle
- Schrödinger equation + some simple quantum systems
- The postulates of Quantum Mechanics *
- Identical particles and exclusion principle
- Is Q.M. really necessary? *

Classical Mechanics

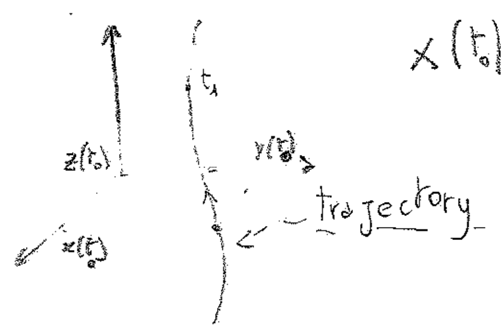
Kinematics gives the rules for describing the motion of objects (it does not predict what the motion will be).

In general it is rather complicated to describe the motion of a real object. Think about a car: the wheels spin, turn, while other parts have different motions.

Abstraction: the point particle object. No internal parts.

At the time t_0, its position is entirely specified by 3 numbers

$$x(t_0) \;,\; y(t_0) \text{ and } z(t_0).$$



definitions $\begin{cases} \text{velocity } v_x = \dfrac{dx}{dt} \text{ etc.} \\ \text{acceleration } a_x = \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2} \text{ etc.} \end{cases}$

During the motion (of a point-like object) the 3 positions will vary and, mathematically, they are just 3 functions of t.

The concept of point-like objects is very useful for at least 3 reasons.

First it can be a very good approximation for more complicated objects. We use this kind of approx in the day-to-day life:

- That car is 300 meters away: 300 m, 3 m, Almost a point. Error of $\frac{3}{300} = \frac{1}{100}$ 1%
- Even the entire earth can be thought as a point when we're dealing with astronomical distances

$r_e = 6400$ Km   Average distance from the sun $1.50\ 10^{11}$ m

The other motivations to study point-like objects later... $\Big\}$ Elementary particle (no size), Center of Mass $\boxed{\mathrm{IA}}$

__Dynamics__ gives us the rules to predict the motion of an object.

__1st Newton law__: In an inertial frame, A particle acted on by no net force (like an isolated particle) moves with constant velocity ($\Rightarrow$ zero acceleration)

__2nd Newton law__: The acceleration of a particle is proportional to the net force

$$\vec{F} = m\,\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

, constant of proportionality is called mass and its a defining feature of the particle. (Various particle will have different masses)

The combination $p = mv$ is very important and is called momentum.

__3rd Newton law__: If A exerts a force on B ($F_{A\to B}$), then B exerts an opposite force on A $F_{B\to A} = -F_{A\to B}$

General feature of Forces

[True for the Gravitational force!]

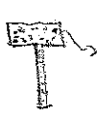
A $\longrightarrow$ $F_{B\to A}$ $\qquad$ B $\longleftarrow$ $F_{A\to B}$

Two consequences of Newton's law

- Consider an isolated system of particles (no external forces). Then the total momentum is conserved. For a 2 particle system

$$\frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \vec{F}_{B\to A} + \vec{F}_{A\to B} = 0 \Rightarrow \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = 0$$

- We can think of a rigid body as many particles held together by strong complicate binding forces

hammer  i^{th} particle

Apply 2nd law to each particle $\vec{F}_i = m_i\vec{a}_i = m_i\dfrac{d^2\vec{r}_i(t)}{dt^2}$

$$\vec{F}_i = \vec{F}_i^{\,Ext} + \vec{F}_i^{\,Bind}$$

Now sum up the forces of all particles $\sum_i$

2nd

1) Use the 3rd law $\sum \vec{F}_i^{Bind} = 0$!

2) Define the center of mass $\vec{R}$

$$\frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{\sum m_i \vec{r}_i}{M} = \vec{R}$$

$M \rightsquigarrow$ Total mass

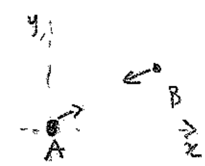
$$F = \sum_i \vec{F}_i^{Ext} = \frac{d^2}{dt^2} \sum_i m_i \vec{r}_i = M \frac{d^2}{dt^2} \vec{R} \leftarrow$$

2nd reason to introduce the concept of point-like objects. The C.M. behaves like a " " " !

<u>Energy</u>. Actually many types of force can be derived from a function U called potential energy (actually it turns out that the concept of potential energy is much more "fundamental" in Q.M. that the one of force !).

$$F_x = -\frac{dU}{dx} \left(= \lim_{\Delta x} \frac{U(x+\Delta x) - U(x)}{\Delta x} \right)$$

The gravitational force $\left(F_{A \to B} = -G \frac{m_A m_B}{r^2} \frac{\vec{r}}{|r|} \right.$



$\left. U_B(r) = -G \frac{m_A m_B}{r} \right)$ has a potential function.

Let us focus on particle B and compute (1-dimensional to be simple)

$$(F_x) v_x = (m a_x) v_x \Rightarrow F_x \frac{dx}{dt} = m \frac{dv_x}{dt} v_x = \frac{1}{2} m \frac{d v_x^2}{dt}$$

$$\Downarrow$$

$$F_x \, dx = d\left(\frac{1}{2} m v_x^2\right) \rightsquigarrow d\Big(\underbrace{\tfrac{1}{2} m v^2}_{\text{kinetic energy}} + \underbrace{U}_{\text{potential energy}}\Big) = 0$$

$\leftarrow$ Energy conservation!!

$$F_x = -dU = -\frac{dU}{dx} dx$$

Tr

The Harmonic Oscillator

We said that macroscopic objects are actually made of many "particles" (atoms or molecules) strongly held together. The forces binding together the particles are complicated, but we can make a simple and <u>very</u> effective approximation.

For sake of simplicity consider an unrealistic 1Dimensional cristal

...  ..

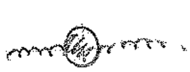
Atoms

These are <u>not</u> real springs. I draw them just to remind us that there are forces keeping the atoms at a fixed distance from each other.

The equilibrium position will be as in the picture above, with each atom at a fixed distance from the next one. A more realistic situation is the one where the atoms move around the equilibrium positions

Atoms in motion. (focus only on <u>chaotic</u> motion, no coherent waves)

Let me focus on one such atom (approx: neglect the fact that the atoms are coupled).



x

x_e ← equilibrium position

No force exerted on the atom

F

x

x_e ← equilibrium position

A force $F = -\dfrac{dU}{dx}$ exerted on the atom

So the potential energy for this atom has to be of this form

$$U(x) = \underbrace{U_0}_{\text{constant}} + \cancel{U_1 (x - x_e)} + \frac{1}{2} U_2 (x - x_e)^2 + \frac{1}{3!} U_3 (x - x_e)^3 + \dots$$

constant — I can neglect the constant $\left(F = -\dfrac{d}{dx} U\right)$

U_1 must be zero

I neglect this and all following terms because $(x - x_e)$ is usually small

Thus the Newton equation for our atom is

$$-U_2(x-x_e) = -\frac{dU}{dx} = F = m\frac{d^2x}{dt^2}$$

<u>Comment</u>: Notice that if $x-x_e$ is small we do not need to know the exact form of the (complicated) force between the atoms but just the coefficient U_2 (usually indicated by K)

Now some math. We need to solve the Eq.

$$\frac{d^2x}{dt^2} = -\left(\frac{U_2}{m}\right)x$$

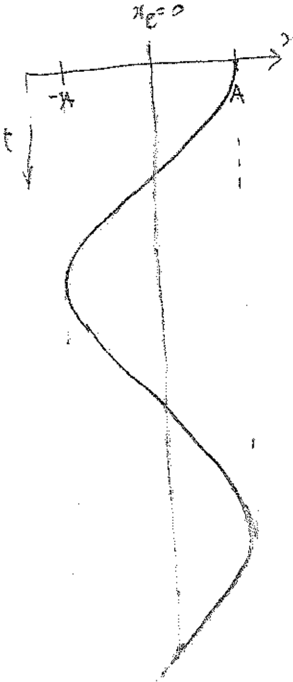
where I fixed the origin of the axis x to coincide with the equilibrium position (just to keep the notation simple).

The solution is $x(t) = A\cos\left(\sqrt{\frac{U_2}{m}}\,t + \phi\right)$

Constants w.r.t. the time t.

<u>Comments</u> 1) U_2 and m are both positive

2) A and ϕ are constants with the following meaning



↳ A is the maximal displacement of the atom from the equilibrium position

because $\boxed{-1 \le \cos(\quad) \le 1}$

↳ $A\cos\phi = x(t=0)$ is just the position of the atom at the instant we decided to call zero. For simplicity we can put

$x(t=0) = A \Rightarrow \phi = 0$

3) $\omega \overset{def}{=} \sqrt{\frac{U_2}{m}}$ is called angular frequency.

$f \overset{def}{=} \frac{\omega}{2\pi}$ "normal" frequency (which means how many times the atom passes at the position $+A$ in a sec)

$T \overset{def}{=} \frac{1}{f}$ period (The time needed to have a complete oscillation)

check: let's check that f has dimensions $\sec^{-1}$ (and so T has dimensions sec - appropriate for a time interval!)

$$[U] = Kg \frac{m^2}{\sec^2} \Rightarrow [U_2] = \frac{Kg}{\sec^2} \Rightarrow \left[\frac{U_2}{m}\right] = \frac{\cancel{Kg}}{\sec^2} \frac{1}{\cancel{Kg}} \Rightarrow [\omega] = \frac{1}{\sec}$$

With this explicit solution we can answer many types of questions.

Ex 1) What is the velocity of the atom at the points of maximal displacement (like $t=0, \frac{T}{2}, T, -T, \ldots$ in our example)?

$$v(t) = \frac{d}{dt} x(t) = -A \sqrt{\frac{U_2}{m}} \sin\left(\sqrt{\frac{U_2}{m}}\, t\right) \quad \text{so}$$

$$v(0) = 0 \qquad v(T) = -A\omega \sin\left(\omega \frac{2\pi}{\omega}\right) = 0 \qquad v\left(\frac{T}{2}\right) = -A\omega \sin \pi = 0$$

always zero!

Ex 2) What is the average position of the atom?

$$\frac{1}{T}\int_0^T x(t)\, dt = \frac{1}{T}\int_0^T A\cos(\omega t)\, dt = \frac{A}{\omega T} \sin \omega t \Big|_0^T =$$

$$= \frac{A}{\omega T} \sin \omega T - 0 = 0 \; (= x_e)$$

$\Rightarrow$ On average the atom is at the equilibrium position!

Mini review of special relativity
(and limits on the validity of Newtonian mechanics)

- There is a limiting speed c ($\sim 300'000$ Km/s $= 3\times10^8$ m/s)
  - No object can go faster than c
  - The light propagates at speed c (in the vacuum).

- There is a very special function entering in relativistic computations

Remember $v^2 = v_x^2 + v_y^2 + v_z^2$

$$\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$ (Sometimes v/c is called β)

- The relativistic energy of a particle is $E = \gamma m c^2$ "rest mass" constant w.r.t. the velocity
  with $m_{rel} = \gamma m \Rightarrow E = m_{rel} c^2$
- The relativistic momentum for a particle is $\vec{p} = \gamma m \vec{v}$

The combination $E^2 - p^2c^2$ is very important...

$$\gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 = \cancel{\gamma^2} m^2 c^4 \left(1 - \cancel{\frac{v^2}{c^2}}\right) = m^2 c^4 \quad E^2 - p^2c^2$$

... because it is an invariant! ($\Rightarrow$ Any inertial observer will find the same result when s/he computes $E^2 - p^2c^2$ for the observed particle)

$$\gamma(v) \xrightarrow[v/c \ll 1]{} 1 + \underbrace{\frac{1}{2}\left(\frac{v}{c}\right)^2}_{\text{very small}} + \underbrace{\frac{3}{8}\left(\frac{v}{c}\right)^4}_{\text{very-very small}} + \dots$$

$$\vec{p}_{rel} = \vec{p}_{New} + O\left(\frac{v^2}{c^2}\right) \qquad \boxed{E_{rel} = \underbrace{mc^2}_{\text{rest energy}} + \underbrace{\frac{1}{2} m v^2}_{\text{Newtonian kinetic energy}} + \underbrace{O\left(\frac{v^2}{c^2}\right)}_{\text{small correction}}}$$

$\Rightarrow$ So we can estimate the error we do by using Newton's mechanics by computing the number $\frac{v^2}{c^2}$. For a F1 car $\beta^2 = \frac{360\ \text{Km/h}}{3\times10^5\ \text{Km/s}} \sim \frac{10^{-1}\ \text{Km/s}}{3\times10^5\ \text{Km/s}} \sim 3\times10^{-7}$!!

The opposite approximation is the so-called super-relativistic approximation, where $\gamma >> 1$. This means that we can neglect the rest mass with respect to the "real" kinetic energy

$$E = \gamma mc^2 = \underbrace{(\gamma-1)mc^2}_{\substack{K \\ \text{kinetic energy}}} + \underbrace{mc^2}_{\text{rest mass}} \qquad \boxed{E \approx pc \ (\approx K)}$$

This implies that massless particles (photon) are always super-relativistic

A unit of energy that is appropriate for particle physics

Usually we measure energies in Joule

$1J$ = work done by a force of 1 Newton in 1 meter

To give you a better idea of what $1J$ is think that the kinetic energy of a whole person that is walking is around $40J$
Way to big unit for particles !! What is usually used is

$1eV$ (electron/Volt) = energy 1 electron has gained after falling through a potential difference of 1 Volt

$$\boxed{1eV = 1.6 \times 10^{-19} J}$$

Then it is also customary to measure

- the momenta in units of eV/c ← speed of light
- the masses in units of eV/c^2

Finally we can conclude the lecture with our first study of a really interesting physical system. Consider a gas at a temperature T and heat it up with the "quantity of heat" Q. The temperature will vary according to the law

$$Q = m\, c\, \Delta T$$

m: Total mass of the gas
c: Specific heat or heat capacity
ΔT: Variation of the temperature

In particular, if the volume is kept constant in the process, the we will use C_V (specific heat at constant volume).
C_V is a different from gas to gas (but some gases have the same C_V). Can we understand why? We need two pieces of information from the kinetic theory of gases (or statistical mechanics)

monoatomic gas

- The temperature is (nothing else than) proportional to the mean (kinetic) energy of the atoms of the gases: higher the $\langle E \rangle \Rightarrow$ higher the temperature. The coefficient of proportionality (if we measure the temp. in degree Kelvin = the Celsius centigrade scale, but with $0°C = 273.16\,°K$) is the Boltzmann constant k

$$k = 1.38\ 10^{-23}\ J/°K \simeq 8.6\ 10^{-5}\ eV/°K$$

and to be precise $\langle E \rangle = \frac{3}{2} k T$

The factor of 3 above is put by convention but in fact it's very handy.

$$\langle E \rangle = \langle \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2 \rangle = \langle \frac{1}{2} m v_x^2 \rangle +$$
$$+ \langle \frac{1}{2} m v_y^2 \rangle + \frac{1}{2} \langle m v_z^2 \rangle = \frac{3}{2} \langle m v_x^2 \rangle$$

The last step relies on the following physical argument: there is no reason why the $\langle v_x^2 \rangle$ should be different from $\langle v_y^2 \rangle$ or $\langle v_z^2 \rangle$. So we can say that each possible direction of motion contribute $\frac{1}{2} KT$ to the average kinetic energy. Now consider a big number of atom of a monoatomic gas like He (helium): for instance an Avogadro number of atoms N_A (Chemists like this number $N_A = 6.02 \times 10^{23}$ and when we have N_A atoms of He they say they have 1 mole of He). The heat capacity of this system is

$$C_V^{1\,\text{mole He}} = \frac{dQ}{dT}\Big|_V \overset{1^{st}\text{ law of thermodinamics}}{=} \frac{dE_{int}}{dT} = \frac{d\langle E \rangle}{dT} = \frac{3}{2} N_A K = \frac{3}{2} R$$

$$R = N_A K = 8.31 \frac{J}{°K \text{ mole}}$$

which agrees with experiment!! see table 18.1 of Young-Freedman

(other noble gas have almost the same C_V !!)

Can we predict the heat capacity of other gases? Take for instance the Iodine I_2, microscopically it looks like

: it's a diatomic gas

We use our previous intuition: the kinetic energy stored in one mole of gas is $\frac{1}{2} N_A k T = \frac{1}{2} RT$ for each possible independent way the particles can move. For I_2 we have

$$6 \times \frac{1}{2} RT = 3RT$$

3 v_x, v_y and v_z of the first atom I; 3 v_x, v_y and v_z of the second atom

alternatively $\Leftrightarrow$
- 3 "velocities" of the Center of Mass of I_2
- 2 independent rotations
- vibrations along the axis connecting the two atoms

- In addition we have the potential energy stored in the "spring".
  We know that for an harmonic oscillator the average kinetic energy $\underbrace{}_{\text{of the vibration}}$ is the same as the average potential energy! (see exercise). So the total (kinetic + potential) energy for our mole of Iodine

$$C_v(I_2) = \overset{\text{Kin}}{3RT} + \frac{1}{2} RT = \frac{7}{2} R = 29.1 \frac{J}{°K\,mol}$$

Exp. value $C_v(I_2) = 27.7 \frac{J}{°K\,mole}$ ... good, but not as good as it was for He

- What about other di-atomic gases

$$C_v(O_2) = 21.10 \frac{J}{°K\,mol} = C_v(HI)$$

Hydrogen Iodine

which are way off the theoretical prediction...

... but they are still multiple of $\frac{1}{2}R$

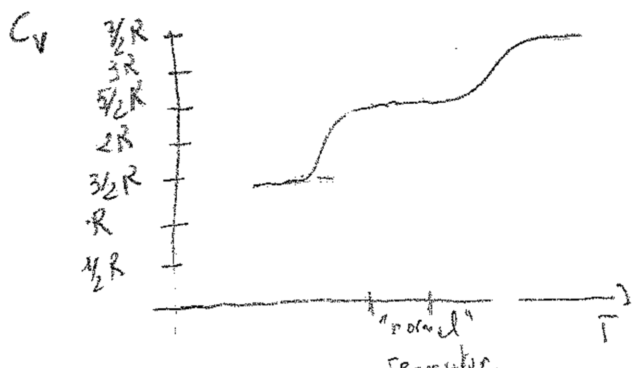
In fact

$$20.7(7) \simeq 5\left(\frac{1}{2}R\right) \quad !!$$

It looks like the atoms can not oscillate ($\Rightarrow$ zero kinetic + potential energy for the oscillation modes!).

How's that possible? [Should have we used relativistic mechanics instead? No see ex.!]

- Even worse for our theory: experimental data shows that C_V varies with the temperature! And, for instance, has a shape of this type



How can we explain this?

Use Quantum mechanics!

This was the first sign of the quantum nature of the microscopical world.