Lecture 3 (week 3)

INTERFERENCE

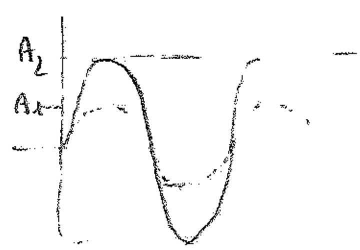
The blackbody radiation problem shows that there is something very fundamental we do not understand about light... we're to find and correct this weak point! So first question is

- Are we really sure that light is a wave?

Young answered this question at beginning of $\overline{XIX}$ century by means of a famous experiments that (apparently!) clarified an issue that was discussed over 2000 years. The basic idea is to exploit the phenomenon of interference (which is nothing else that the value of the intensity of the superposition of two waves).

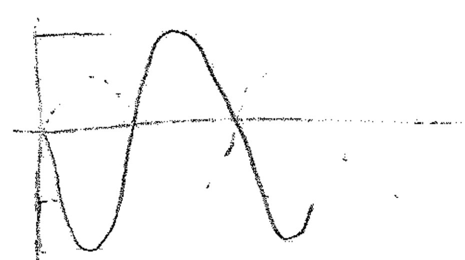
- Constructive interference: $A_1 \sin(kx-\omega t) + A_2 \sin(kx-\omega t)$

The 2 waves add up and make a wider oscillation ←
$(A_1+A_2)\cos(k\cancel{x}-\omega t)$



- Destructive interference: $A_1 \sin(kx-\omega t) + A_2 \sin(kx-\omega t+\pi)$

The 2 waves partially cancel each other and the result displays a smaller oscillation

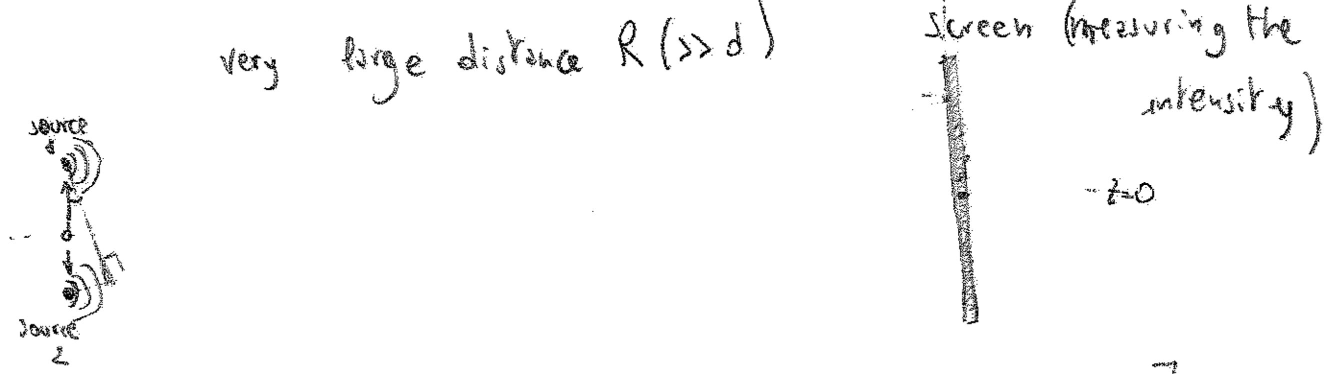


[Notice that if $A_1 = A_2$ the result of the destructive interference is zero, i.e. no wave at all!]

Thus Young's idea was to take 2 source of light and see whether the interference is there or not... however setting up this experiment in the right way is not easy!

- 1st We need to realize that the light-wavelength is related to the colour. Since we want to have nice simple waves of cos (or sin) form and not a complicated superposition of those we need 2 <u>monochromatic</u> sources
- 2nd Suppose we can fix things in such a way that the two sources are exactly coherent (in phase, i.e. the difference of phase $\Delta\phi=0$)

Our set up would be the following



$$\psi_1 = A\cos(\omega t - k\cdot r) \quad [r \text{ distance from the source } 1]$$

$$\psi_2 = A\cos(\omega t - k r') \quad [r' \text{ distance from the source } 2]$$

If light can be represented by waves (ψ_1 and ψ_2), the intensity seen on the screen should have a very funny behaviour let us see why:

1) The distance between the point $z=0$ and the source 1 is <u>equal</u> to the distance between the same point ($z=0$) and the source 2. [$r=r'$ for $z=0$]

Thus for $z=0$ $\quad \varphi_1\big|_{z=0} = \varphi_2\big|_{z=0}$ and the two waves interact in a constructive way

2) Let us move up on the screen $(z>0)$. Now the two distances (r and r') are not equal anymore. If $R \gg d$ we can approximate $r'-r \sim d \sin\vartheta$ and $k(r'-r) = \frac{2\pi d \sin\vartheta}{\lambda}$ tells us how much the waves are out of phase ($\frac{2\pi d \sin\vartheta}{\lambda}$ is what previously we called $\Delta\phi$). So if $\Delta\phi = 2\pi n$ $(n=0,\pm1,\pm2,\dots)$ the two waves are again in phase and are constructive

constructive interference	$d \sin\vartheta = n\lambda$

If $\Delta\phi = \pi(2n+1)$ $(n=0,\pm1,\pm2\dots)$ the two waves are just opposite and we get a destructive interference

destructive interference	$d \sin\vartheta = \left(n+\frac{1}{2}\right)\lambda$

$\Rightarrow$ So we expect to see on the screen (the z-axis) a succession

1) of bright and dark bands (interference fringes).

2) Moreover we expect that the brightness (intensity) in the point of constructive interference is 4 times bigger than the one we would get with 1 source (remember that $\langle I\rangle \sim \langle\varphi^2\rangle$!)

But we can do more than that! We can relate the wavelength and the distance between the center ($z=0$, the brightest fringe) and the other bright points

$$\frac{\Delta_m}{R} = \tan\vartheta'_m \Rightarrow \Delta_m = \tan\vartheta'_m R$$ ... but ϑ' is almost ϑ (plus small corrections $\sim d/R$). Also $\vartheta \ll 1$ so that $\sin\vartheta \sim \tan\vartheta$ (again plus small corrections $\sim (d/R)^{2}$). So we have

$$\boxed{\sin\vartheta_m = \frac{\Delta_m}{R}}$$

By using this in our previous eq. we get:

$$\boxed{\lambda = \frac{d\,\Delta_m}{R\,m}}$$

Typical Young experiment with light has

$R = 1\ m$ $\quad d = 2\times10^{-4}\ m = 0.2\ mm$

and $\Delta_3 = 7.5\ 10^{-3}\ m = 7.5\ mm$

$$\Rightarrow \lambda = \frac{2\cdot10^{-4}\times7.5\ 10^{-3}}{3} = 5\times10^{-7}\ m$$

The light used is blue!

It is not so easy to get two coherent sources!

- They must be small (otherwise diffraction ... see next week!)
- They must be exactly in phase! Young's idea