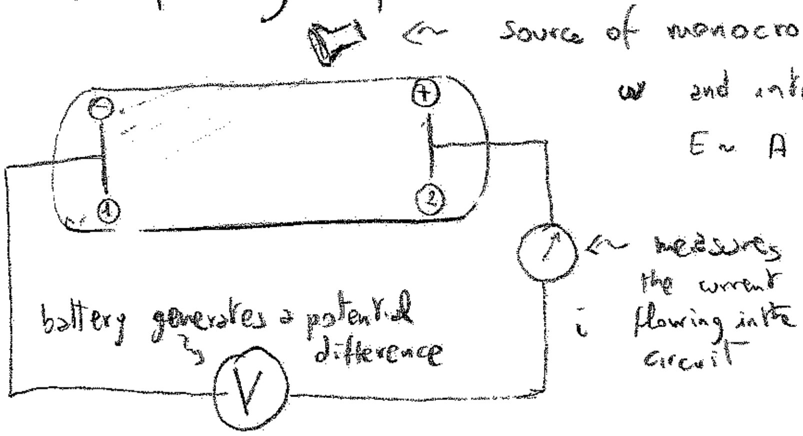
PHOTO ELECRIC EFFECT

Through the Young experiment we (thought we) proved the wave-like nature of light (we even computed its wavelength!) ... now I will give you an experimental proof of the opposite (!): light is made out of small point-like particles !!

Consider the following experiment



$E \sim A \cos(\omega t - kr)$ $\quad \frac{\omega}{k} = c$

Quantities in the game

1) The Voltage $\boxed{1}$ given by the battery

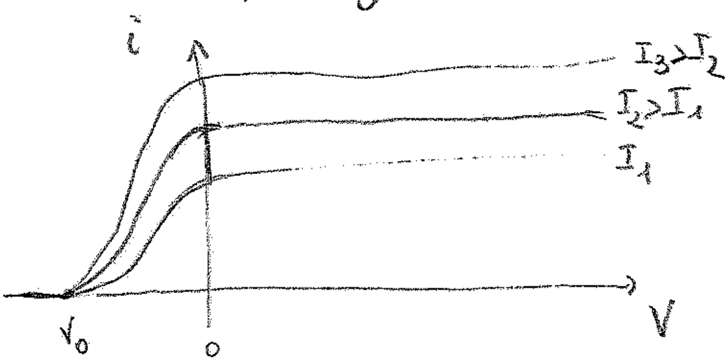
$V > 0$ Plate ① is effectively the cathode (repels negative charged objects)

$V < 0$ Plate ① is the anode

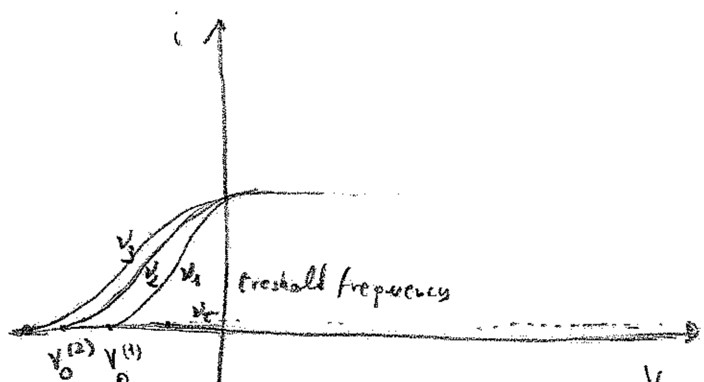
2) The intensity $\boxed{2a}$ $(I \sim A^2)$ of the light and its wavelength $\boxed{2b}$ (or frequency)

3) i $\boxed{3}$, the current flowing in the circuit

Two puzzling results:



The frequency ν of the incident light IS FIXED



The intensity I of the incident light IS FIXED

Some of these observations fit naturally with our idea that light is a wave

1) i (current) $\sim$ I (intensity of the light) $\Leftarrow$ more energy pumped in
more electrons are extracted

2) i increases with V, but then stops (when all electrons emitted reach the opposite plate the current i can not increase more!)

Others observations are just incredible

1) The existance of a treshold frequency ν_t (no current produced if the light is below ν_t regardless of what I is!!)

2) V_0 (stopping potential) is independent of I

3) V_0 depends (linearly!) on ν. Why?

Rivolutionary proposal by A. Einstein (1905):
The energy carried by a wave of light (and actually any E.M. wave) is not continuously distributed by it is concentrated in lumps or packages or quanta (!).
Now we start understanding point 1. It's true that the total energy of the wave is related to its intensity, but this energy should be thought as the energy of each quantum (E_q) multiplied by the number of quanta present in the wave. If E_q is not sufficient to extract the electron from the atom we will see no current independently from the number of quanta hitting the atom

So the proposal of Einstein was

$$\underbrace{E_q}_{\text{energy of each quantum}} \sim \underbrace{\nu}_{\text{frequency}} \qquad\qquad I \sim \#\text{ of quanta}$$

The idea was not completly new! M. Planck 5 years before (1900) proposed something similar to explain the blackbody radiation. What Planck proposed that the interaction between matter (the blackbody) and radiation was such that the emissions of light from the blackbody could happen only in discrete lumps (or quanta!). Again the energy of each of this quanta was proportional to the frequency of the emitted wave. The constant of proportionality is universally indicated with h or $\left(\hbar = \frac{h}{2\pi}\right)$

$$E_q = h\nu = h\frac{1}{T} = \left(\frac{h}{2\pi}\right)\frac{2\pi}{T} = \hbar\omega$$

(various alternative and equivalent expressions)

→ h is a new constant of nature (like c!)

$$[h] = \text{Energy} \times \text{second} = Kg\,\frac{m^2}{s} \leftarrow \text{Dimensions of } h$$

Because we can use h in describing the blackbody radiation our dimensional argument about the blackbody

radiation intensity $\boxed{R2.14}$ which yields Rayleigh law $I(\lambda) = 2\pi \frac{c\,KT}{\lambda^4}$
is not valid any more!

Planck was able to derive from his quantum hypothesis the following (amazing) result:

$$I_T(\lambda) = \frac{2\pi h c^2}{\lambda^5 \left[e^{\frac{hc}{\lambda KT}} - 1 \right]} \qquad \text{(Planck radiation law)}$$

which agrees spectacularly with the experiment

$$\rightarrow \quad I(\lambda) \xrightarrow{\lambda \gg 1} \frac{2\pi h c^2}{\lambda^5 \left[1 + \frac{hc}{\lambda KT} - 1 \right]} = \frac{2\pi c\, KT}{\lambda^4} \quad \text{Rayleigh law}$$

$$\rightarrow \quad I(\lambda) \xrightarrow{\lambda \ll} \sim \lambda^{-5} e^{-\frac{hc}{\lambda KT}} \rightarrow 0 \qquad \text{(no U.V. catastrophe!)}$$

→ We can derive (!) Stefan - Boltzmann law

$$I = \int_0^\infty I(\lambda)\, d\lambda = \underline{2\pi h c^2} \int_0^\infty \lambda^{-5} \left[e^{\frac{hc}{\lambda KT}} - 1 \right]^{-1} d\lambda$$

$$= 2\pi h c^2 \int_0^\infty \nu^3 \left(e^{\frac{ch}{KT}\nu} - 1 \right)^{-1} d\nu = 2\pi h c^2 \left(\frac{KT}{ch} \right)^4 \int_0^\infty x^3 \left(e^x - 1 \right)^{-1} dx$$

$$= \frac{2\pi K^4 T^4}{c^2 h^3} \left[\frac{1}{240} (2\pi)^4 \right] = \underbrace{\left(\frac{2\pi^5 K^4}{15 c^2 h^3} \right)}_{\text{This is } \sigma \;!!} T^4$$

→ We can derive Wien's law $\lambda_m T = \text{const.} = 2.90\ 10^{-3}\ m\,°K$

λ_m is such that $\frac{dI}{d\lambda} = 0 = -5 + \left[e^{\frac{hc}{KT\lambda_m}} - 1 \right]^{-1} \frac{hc}{KT\lambda_m} \Rightarrow T\lambda_m = \frac{hc}{Kx}$

where x solves $5e^x = 5 - x$ $\quad x \sim 4.965$

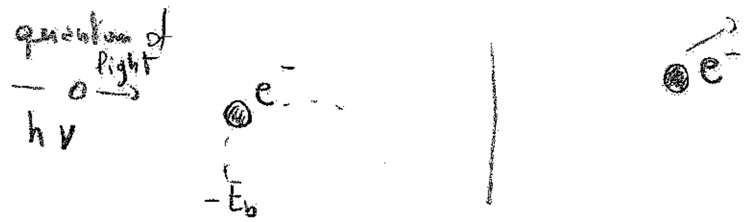
Einstein took Planck's suggestion and applied to the nature of light itself (it wasn't just the emission of light that was quantized, but the energy of the wave itself!). So

$E = h\nu \Rightarrow$

1) We have a current in the photo-electric circuit only if $\nu > \frac{E_b}{h}$, where E_b is the binding energy of the electron inside the atom

2) If $\nu > \frac{E}{h}$ $\Rightarrow$ the extracted electron comes out with a non-zero kinetic energy!

Think about the process as a billard collision (!)



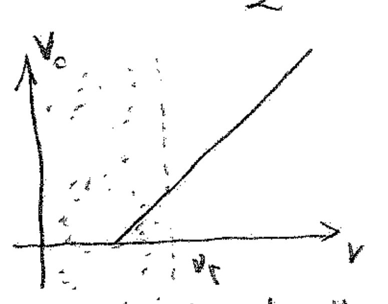
$\frac{1}{2} m_e v^2 = h\nu - E_b$

[called "work function" ϕ on the book]

We can measure the kinetic energy of the outgoing electron by see what the stopping potential V_0. If $|eV| \geq \frac{1}{2} m_e v^2$ then no electron will reach the other (negative charged) plate. So the maximum kinetic energy of the "emitted" e is

$$eV_0 = \frac{1}{2} m_e v^2 = h\nu - E_b$$

Plot V_0 in function of ν



The slope of this line is $\frac{h}{e}$

cut out because it's below the treshold frequency

We have two distinct ways to compute the value of h

- Use Planck's law of Blackbody radiation
  [For instance measure σ and the derive h]

- Use the photo-electric effect
  [Compute the slope of the $V_0(\nu)$ line]

Millikan performed high precision ((for the time) experiment comparing the results obtained in the two cases. His hope was to find two different results so that I could kill Einstein's idea of the quanta of light (or photons) ... instead he found the same number:

$$\boxed{h = 6.626 \; 10^{-34} \; J \cdot s}$$

$h = 4.14 \; 10^{-15}$ eVs

$\hbar = 6.59 \; 10^{-16}$ eVs

He confirmed Einstein theory (and he got also the Nobel prize!).

Let me go back to the new relation we found $E = h\nu$ and combine it with something we already now from our understanding of waves: $\nu \cdot \lambda = c$ (or $\frac{\lambda}{T} = \frac{\omega}{k} = c$; from $A\cos(\omega t - kx)$ and the fact that c is the speed of light). Then

$$E = h\nu = h\frac{c}{\lambda} = \frac{h}{\lambda} c$$

This is a relation we already found in special relativity!
The relation between energy and momentum for a massless particle (i.e. a particle that can not be put at rest => it has no rest mass) is

$$E = pc$$

Together with

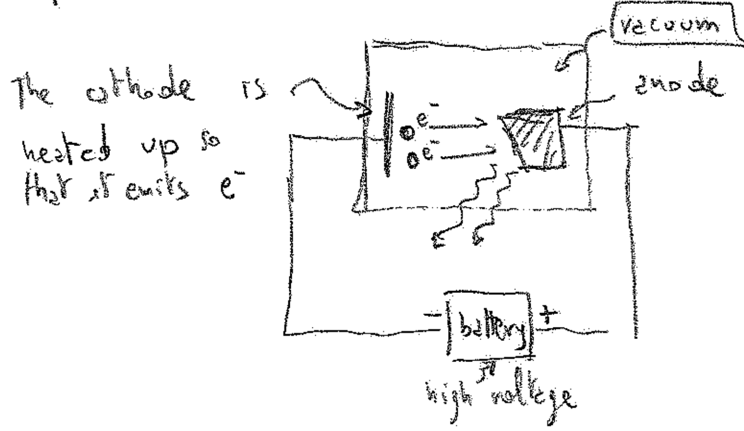
$$E = \frac{h}{\lambda} c \quad \Rightarrow \quad p = \frac{h}{\lambda}$$

Not only does the quantum of light (the photon) has some energy but it also has a momentum - Summarizing

$$\boxed{E = h\nu = \hbar\omega \quad \text{and} \quad p = \frac{h}{\lambda} = \hbar K}$$

→ These relations hold for any electromagnetic-wave (any ω) and seem to say that light really has a particle-like behaviour (it carries both energy and momentum!). This is even more evident when we increase the frequency of E.M. (or shorten the wavelength to values smaller than 10^{-9} m). This type of E.M. radiation was studied first by Rötgen in 1895 who called it X-ray because he was not sure that it was really dealing with some E.M.-wave. He was puzzled by the fact that apparently the X-rays could pass through many materials (and there was some partial absorption only if the material was dense). Let me sketch for you the apparatus Rötgen used to produce the first X-rays. It again exploit the fact that it is possible to extract electrons from a metal by pumping in it energy - He used thermal energy

(a process called thermionization)



The emitted electrons are accelerated by the potential difference which is big... so they gain a large kinetic energy till they hit the anode.

When the electrons reach the anode two different processes may happen:

1) They're slowed-down by the collisions with the atoms of the anode.
$\Rightarrow$ The e^- are decelerated $(a \neq 0)$ $\Rightarrow$ They emit EM waves (or photons). What's the maximal frequency of the emitted X-rays?

We can answer again by using $E = h\nu$!

The electron is released from the cathode with some kinetic energy (like in the photo-electric effect), but it is then accelerated by the high voltage gaining much more energy than it had at the beginning. So we can neglect the initial energy and think that before reaching the anode the energy of the electron is just $e\Delta V$. Then if it is stopped in one single collision it can emit just 1-photon which will carry all its energy (limit case!). Then by energy conservation:

$$\boxed{e\Delta V = h\nu_{emitted}^{max} = h\frac{c}{\lambda_{min}}}$$

bremsstrahlung limit

Notice λ_{min} does not depend on material used to make the cathode.

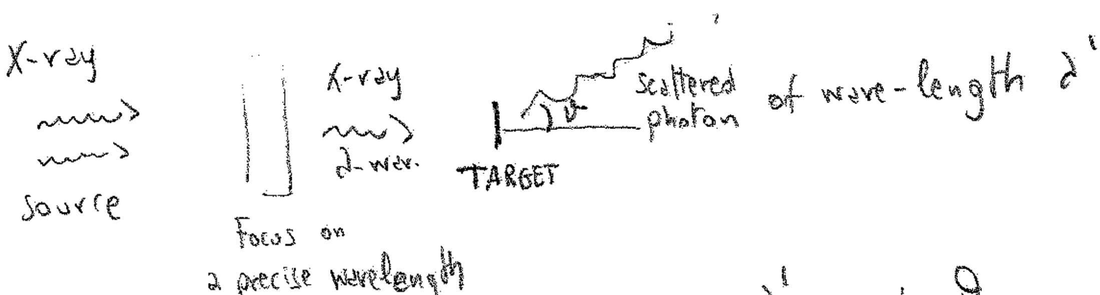
2) A second possibility is that when the electrons reach the anode they do transfer part of their energy to the

atoms of the anode (and excite them). These "excited" atoms then decay to their original (equilibrium) state and again emit highly energetic photons.

The main difference between the X-rays emitted through the process 1) and the process 2) is that in the first case we have a continuous spectrum (various frequencies are emitted, varying ν continuously) while in the case 2) the spectrum it's discrete (only particular freq. are emitted).

COMPTON SCATTERING

Let us now use the X-ray in an experiment that was first explained by Compton which strikingly shows the particle-like properties of the E.M. radiation. Very schematically:

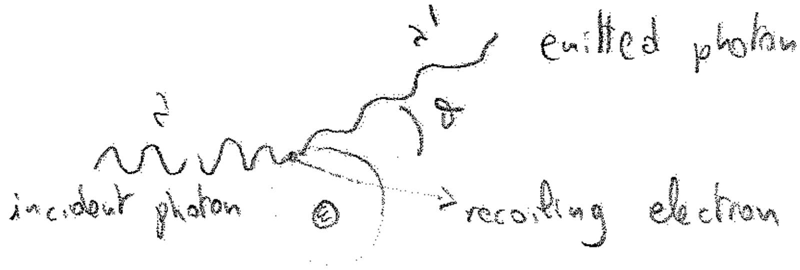


We want to study the relation between λ' and ϑ.

Again when the X-ray hit the target two different kinds of processes may happen

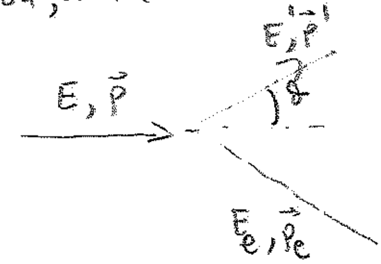
1) The incident photo is absorbed by an atom of the target which then go back to its original state by emitted a photon

2) The incident photon (in the X-ray) interact just with an electron in the atom and the electron recoils away from the atom because of the collision



We're interested in this second type of events! Let us think that the process above can be described exactly as the collision between billard balls. The only thing we have to be careful about is that, since energies in the game are rather big, velocities can be big too ... so we should use <u>relativistic kinematics</u>.

a) We know that the incident photon carries an energy and a momentum $E = \frac{h}{\lambda} c$ and $p = \frac{h}{\lambda}$. Analogously the emitted photon has energy $E' = \frac{h}{\lambda'} c$ and momentum $p' = \frac{h}{\lambda'}$. The energy E_e and the momentum p_e of the recoiling electron should just complete E' and p' to give the values $(m_e c^2 + E), (p)$ so to have energy and momentum conservation (notice that we neglect the kinetic and potential energy of the electron before the collision, since it is small compared to the other energies).



$$E + m_e c^2 = E' + E_e \quad (1)$$

$$\vec{P} = \vec{P}' + \vec{P}_e \quad (2)$$

Rewrite ① and ②

$$E - E' + m_e c^2 = E_e \qquad \xrightarrow[\text{two relations}]{\text{square this}} \qquad E_e^2 = (E - E' + m_e c^2)^2$$
$$\vec{p} - \vec{p}\,' = p_e \qquad\qquad\qquad\qquad p_e^2 = (\vec{p} - \vec{p}\,')^2$$

Combine them as follows ① $- c^2$ ②

$$\to \quad E_e^2 - p_e^2 c^2 = (E - E' + m_e c^2)^2 - (\vec{p} - \vec{p}\,')^2 c^2$$

Then let us recall special relativity $\boxed{E^2 - p^2c^2 = m^2c^4}$ valid for any particle of mass m. So the l.h.s. simplifies and we get

$$m_e^2 c^4 = E^2 + (E')^2 - 2EE' + 2E m_e c^2 - 2E' m_e c^2 + m_e^2 c^4 - p^2c^2 - p'^2c^2 + 2\vec{p}\cdot\vec{p}\,' c^2$$

Then use the relation $E = pc$ valid for photons

$$0 = m_e c^2 E - m_e c^2 E' \underbrace{- EE' + pp'\cos\vartheta\, c^2}_{-pp'c^2(1-\cos\vartheta)}$$

$$0 = m_e (E - E') - pp'(1 - \cos\vartheta)$$

$$\frac{E - E'}{pp'} = \frac{1-\cos\vartheta}{m_e} \Rightarrow \frac{c}{p'} - \frac{c}{p} = \frac{1-\cos\vartheta}{m_e}$$

Use again $E = pc$

Now use quantum mechanics $p = \frac{h}{\lambda}$

$$\Rightarrow \boxed{\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\vartheta)}$$

Compton scattering
$1923 \xrightarrow{\text{NOBEL}} 1927$!!!

Question: So can we describe everything through the concept of point-particle ?
(and wave are derived concepts like in the sound ?)

Or maybe ... since $p = \frac{h}{\lambda}$ Then $\lambda = \frac{h}{p}$ and we can see a wave-like behaviour of matter itself ???