Historical experiment exploiting wave-diffraction

We have seen that wave superposition gives rise to very peculiar phenomena (peculiar to wave dynamics and absent in particle mechanics)

- Interference (superposition of two waves $\boxed{R.\ 3.3-3.4}$)

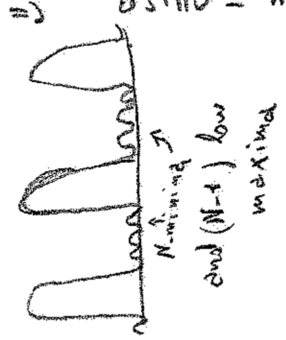
distance between two consecutive bright-fringes $\Delta = R\dfrac{\lambda}{d} \Rightarrow$! λ cannot be too smaller than d if we want to have a visibile fringe-pattern

- Diffraction (superposition of many waves)

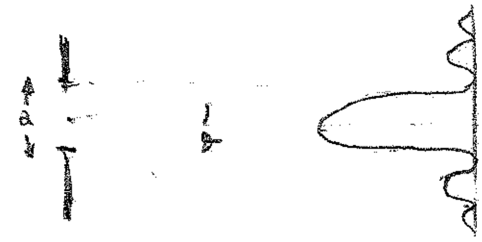
→ Diffraction grating

The difference of phase between the waves with source 1 and those with source 2 is (again) $k\,d\sin\vartheta$; when this is a multiple of 2π we have constructive interference $\frac{2\pi}{\lambda} d\sin\vartheta = 2\pi m \Rightarrow d\sin\vartheta = m\lambda$

On the (far away) screen we see the following pattern (with N sources)



→ Diffraction by obstacles (or slits). Thanks to Huygens' principle we can see a slit as a set of (infinetely) many sources of spherical waves ... and then apply the usual approach of summing up waves arriving from different sources. The result is



The width of the central band is fixed by ϑ_c $\boxed{\sin\vartheta_c = \frac{\lambda}{a}}$

- If $\lambda \ll a$ then $\vartheta_c \ll 1$ => There is very little spreading (!)

The wave behaves almost as an ensemble of particles !!

The intensity of the other maxima (the non-central ones) is rather low => I decreases as the square of $\frac{\pi a \sin\vartheta}{\lambda}$!

Secondary maxima $\frac{\pi a \sin\vartheta_m}{\lambda} \sim \frac{(2m+1)\pi}{2}$ (precise relation is more complicated)

$m = 1, 2, \ldots$ [Notice that there is no maximum for $m=0$, contrary to what one might have guessed!]

$$I_m \sim I_0 \frac{1}{(m+\frac{1}{2})^2 \pi^2}$$

$I_1 \sim 0.045\, I_0$ $\quad I_2 \sim 0.0165\, I_0$ etc.

=> Thus the peculiar nature of wave is most clear when

- two sources are close (interference)
- slits/obstacle are small (diffraction)

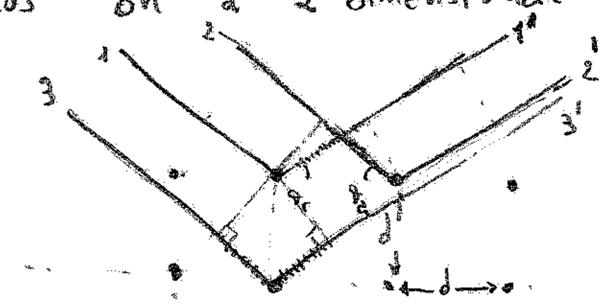
where small/close means not too much bigger than the wavelength of the wave we're considering.

=> This is why it took so much time to see the wave-like nature of light! Visible light $\lambda \sim 3.5-7\ 10^{-7}$ m!
[or in energies $E = h\nu$ few eV $\sim 2-4$ eV]

=> Now you can easily guess that when X-rays were first produced, nobody was really sure that they were an E.-M. radiation ... they looked a bit line particles since their wavelength is so small (1000 times smaller then visible light).

[X-ray $\lambda \sim 10^{-10}$ m or in energies few KeV $\sim 10^3$ eV]

Is there any real possibility to see experimentally the wave nature of X-rays? How can we pack sources so close ($d \sim 10^{-10}$ m) or make slits so small? Quite incredibly the wave-nature of X-rays was tested not much later than their discovery (1895 Rötgen → 1912 Von Laue proposed that <u>cristal</u> can be a perfect diffracting grating for X-rays). For simplicity let us focus on a 2-dimensional cristal



d should be of the order of the atomic size (think of what happens when you try to pack many spheres in a box).

We know that the atomic size $\sim 10^{-10}$ m !

In fact for salt NaCl $d \sim 2.82 \; 10^{-10}$ m

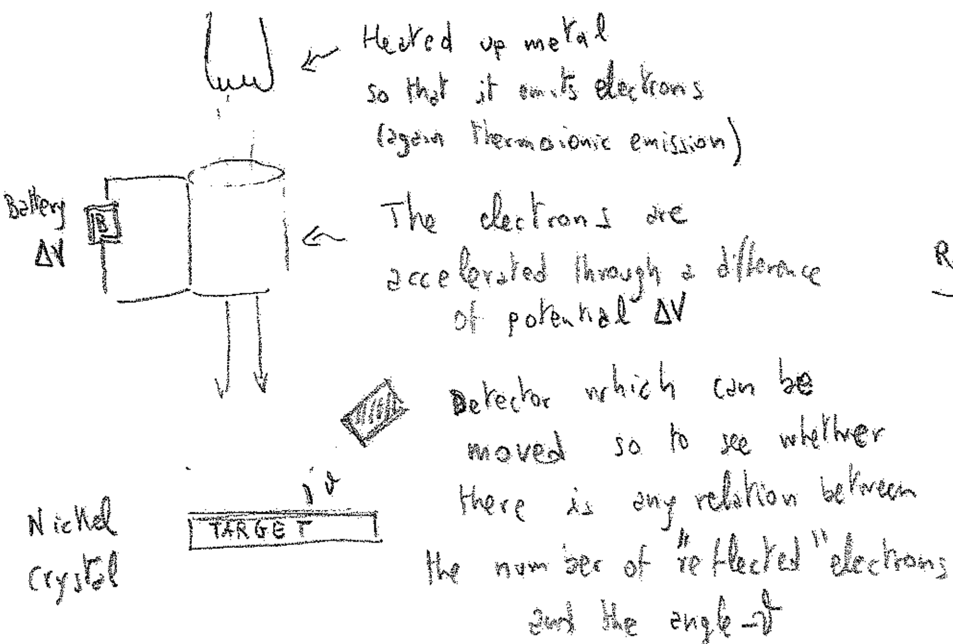
It looks good! Let us see what happens when an X-ray is reflected by the cristal surface

- If $\vartheta_a = \vartheta_r$ then the lengths of ray 1-1' and 2-2' are exactly equal and so they are in phase (constructive interference)

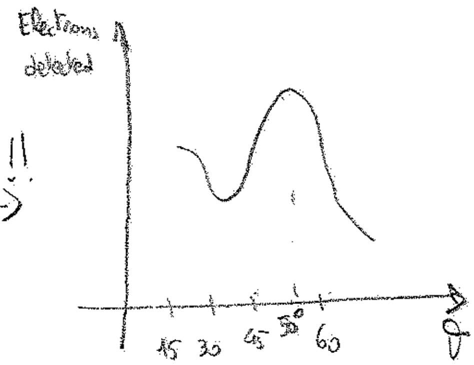
- Ray 3-3' travels for a longer path than 1. In particular this path is $2d\sin\vartheta$ longer (see the ~~###~~ line of 3-3' in the figure). Again to have constructive interference we must meet the condition $\boxed{2d\sin\vartheta = m\lambda}$.

Thus for a particular cristal (this fixes d) and a particular "monochromatic" X-ray we have angles giving strong reflected intensity (all atom in the cristal partecipate in a constructive way!). This phenomenon is called $\boxed{\text{Bragg reflection}}$ from William and Lawrence Bragg (shared Nobel prize ... Lawrence was William's son and is the youngest ever nobel prize he was 25 when he went to Stockholm!).

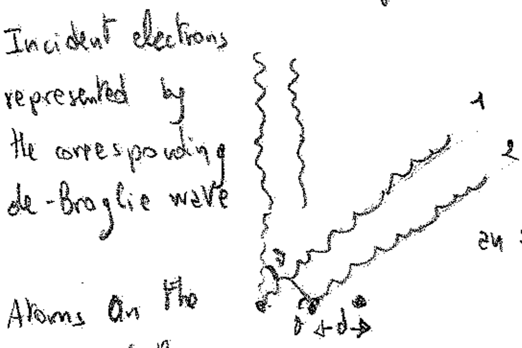
⇒ In 1927 Davisson and Germer at the Bell Telephone laboratories were playing with X-rays, in particular, with an apparatus similar to this one



Result!!



Microscopically the picture is the following



(d= 2.15 $\times 10^{-10}$ for nickel).

reflected electrons are in a constructive interference if the length difference of path 1 and 2 is an integer multiple of the de Broglie wavelength.

$$\Downarrow$$

$$d \sin\vartheta = n\lambda \quad \text{where} \quad \lambda = \frac{h}{p} \sim \frac{h}{\sqrt{2mK}}$$

In the last step I used the non-relativistic approximation. In fact Davisson and Germer used a "battery" with $\Delta V = 54\ V$ so that the kinetic energy of the incident electrons was 54 eV (10^4 times smaller than the rest mass!). So the de-Broglie wavelength for their incident electrons was $\lambda_{dB} = \frac{h}{\sqrt{2me\Delta V}} \sim 1.7\ 10^{-10}\ m$ (good! it's comparable with the crystal spacing!). Then, if electrons are really described by de-Broglie waves, then we should get a constructive interference (and thus a maximum of detected electrons) for

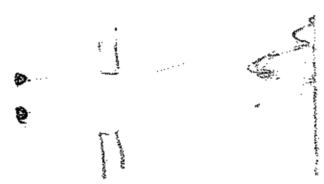
$$d \sin\vartheta = \lambda \quad \Rightarrow \quad \sin\vartheta = \frac{\lambda}{d} \sim \frac{1.7}{2.15} \quad \Rightarrow \quad \vartheta \sim 50^\circ$$

which is exactly what we see !!

Time scale	1925	1927	1929
	De-Broglie proposal	Davisson-Germer experiment	Nobel Prize for De-Broglie

Diffraction and resolving power

The last consequence of diffraction I want to mention is the limit it imposes on the resolution power of optical instruments. The question is: if I have two distinct source of light, when can I see them well separeted? The problem is less simple than it looks like... any optical instrument is made of lenses and slits (including our eye!). We know that when a wave emerges from a slit the border of the wave-ray are blurred by diffraction



The two big bright bands give a blurred image of the emitting source and may overlap

This overlap is more severe when the two sources are close (their distance is of the order of the wave-length) and the blurring is more pronounced when the size of the slit is small. So we can intuitively understand that an optical microscope can not resolve distances smaller than the light wave length $\sim 5\ 10^{-7}$ m ... so apperently we will never be able to see the atoms (whose size $\sim 10^{-10}$ m). In order to do that we would need a wave with much shorter wavelength like the X-rays: the very energetic E.M. radiation whose $\lambda \sim 10^{-10}$ m and whose energy (for each photon) is of order of few KeV. However we now know that there is a better candidate: why don't we use electrons to probe small distances? We've just seen that their de-Broglie wavelength is $\sim 10^{-10}$ m when each electron has only 50 eV of kinetic energy. Thus we can reach the required resolving power in a much easier way.

This discussion on the resolving power seems to have a great importance for experiments only ... but this is not the case! It was the starting point which led W. Heisenberg to the discovery of the uncertainty principle. This and other strangeness of Q.M. will be at the center of the second part of this course. The second part will be a bit more abstract and will have a more "theoretical" approach. Instead of analyzing experiments and derive some rules to understand what is happening we will try to develop a more general framework and answer some questions about the

- Where is a photon in the E.M. wave?
- O.K. de Broglie waves are real ... but what is the stuff they are made of?
- Can I describe a single photon? or a single electron? (and in this case what is the associated wave?)